Modelling Tax Evasion

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Some Tax Concepts

- Avoidance (legally arranging to pay less tax)
- Evasion (illegally not paying tax)

- Tax rate: proportion of income taxed
- *Marginal* tax rate: proportion of the xth dollar which is taxed
- *Progressive* tax: tax rate increases with amount taxed (justified e.g. by utility)
- *Regressive* tax: tax rate decreases with amount taxed


INCOME
TAX
PAY £200


GST
Goods and Services Ta

In models of taxation, restricting to proportional taxes makes things simpler. Real-life examples:

- Religious taxes
- Income taxes in some countries (usually small countries) e.g. Estonia 21%.

Question to ponder:

What is the expected relationship between the proportional tax rate τ and the amount of tax evasion?

Models of tax evasion (some examples)

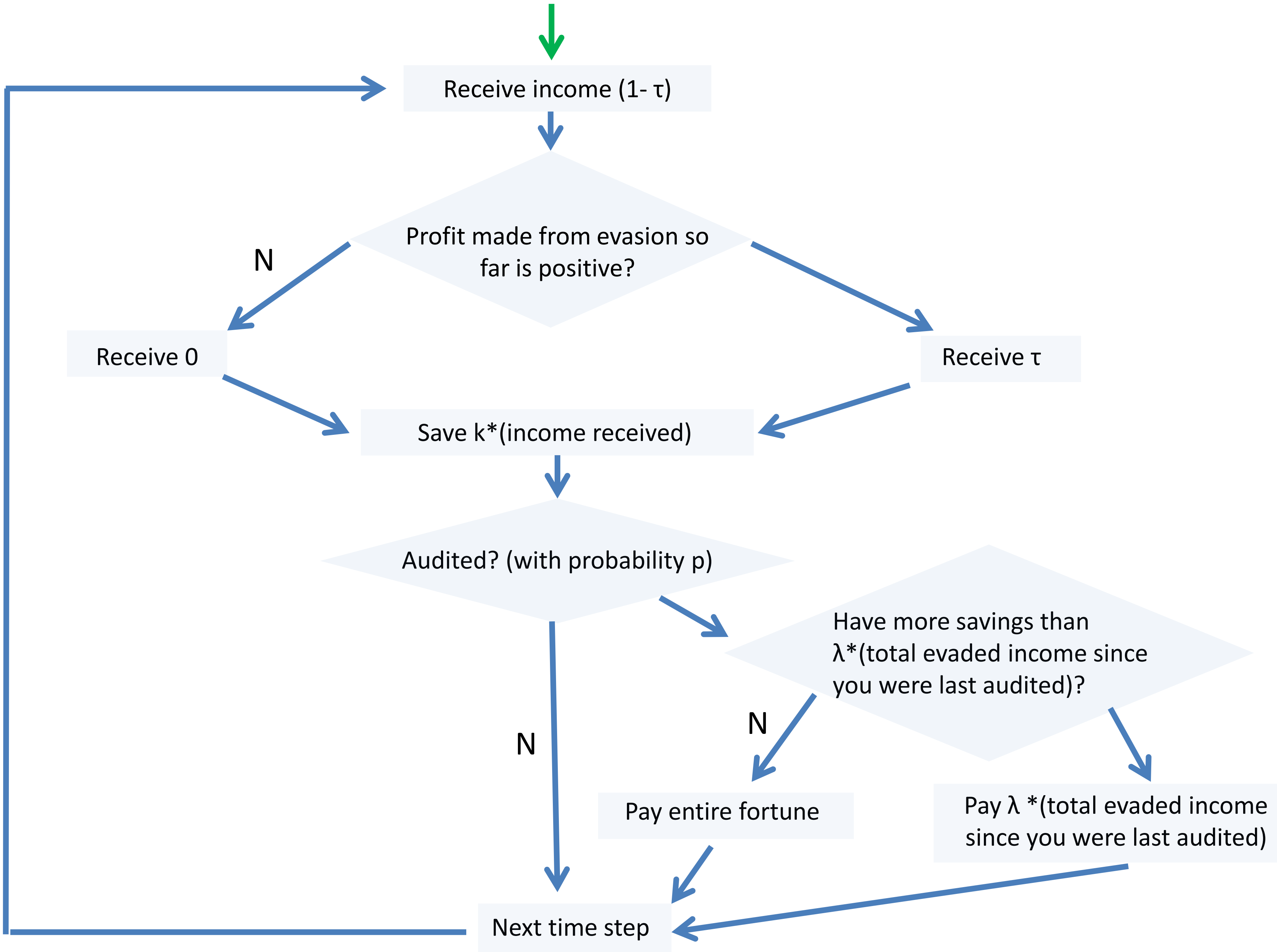
- Allingham & Sandmo, 1972: single taxpayer; static.
- Yitzhaki, 1974: higher tax rates should increase compliance. **Paradox!**
- Klepper/Nagin/Spurr, 1991: includes savings rates of taxpayers.
- Korobow/Axtell/Johnson, 2007: agent-based models; networks; multiple taxpayers.
- Zaklan et. al. 2008: use the Ising model.

Aim

Come up with an agent-based model in which the agents behave simply and which is more realistic than existing models.

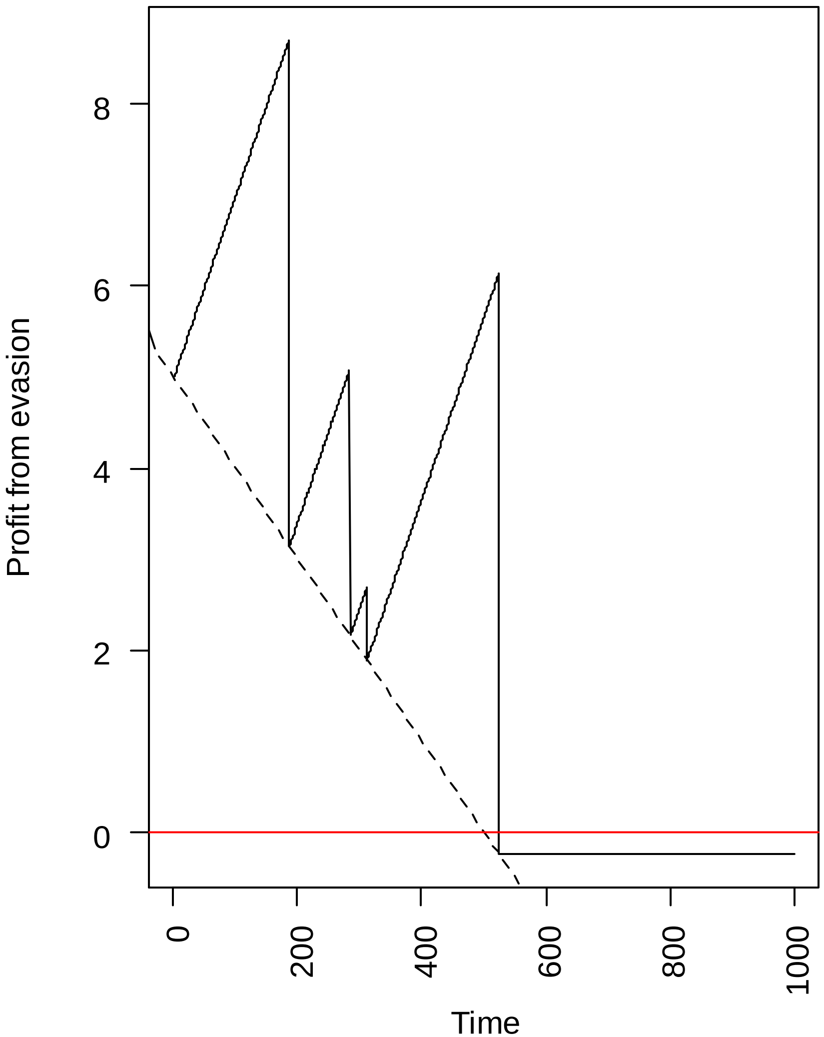
Very simple model of a single taxpayer:

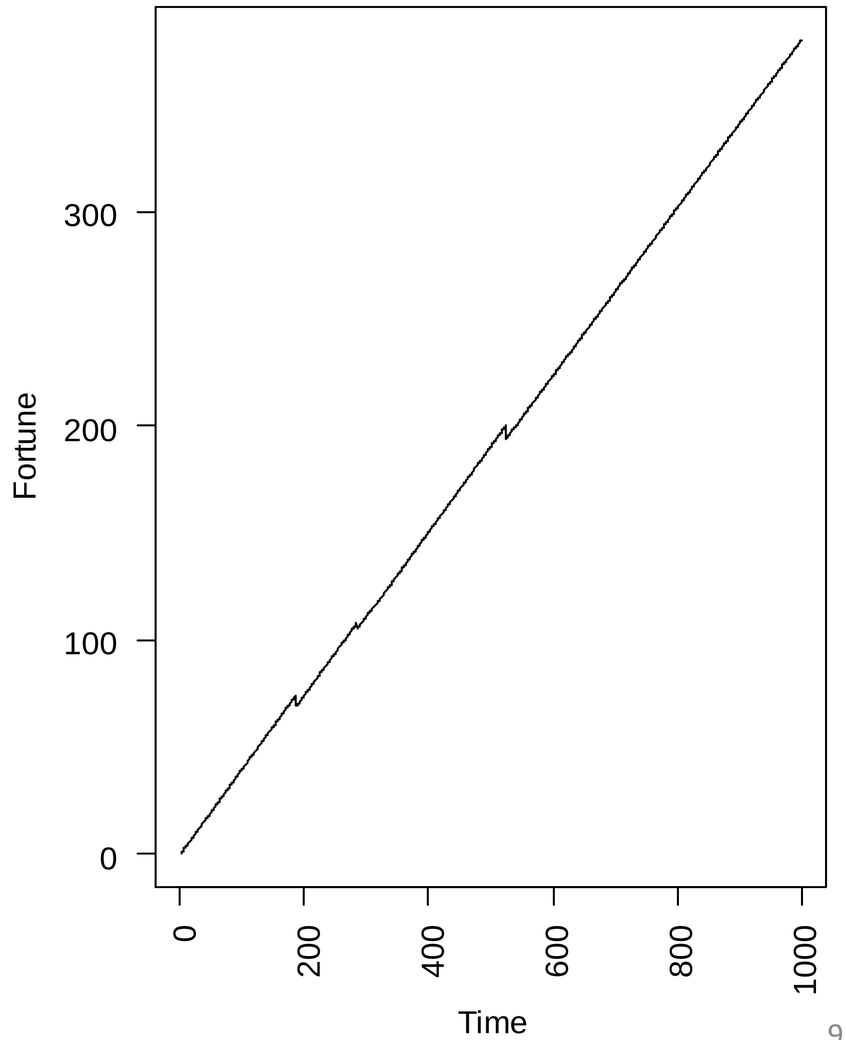
- Fixed income of \$1 per period.
- Fixed proportion k of income which is saved (rest is assumed to be spent.)
- Fixed proportional tax τ.
- Fixed probability of audit p.
- Fixed penalty $\lambda > 1$ that must be paid on total evaded tax when caught.


Receive income (1- τ)
Profit made from evasion so far is positive?
N
Receive 0
Receive τ
Save k*(income received)
Audited? (with probability p)
Have more savings than λ*(total evaded income since you were last audited)?
N
N
Pay entire fortune
Pay λ *(total evaded income since you were last audited)
Next time step

An example with τ = 2%, p = 0.01, λ = 1.5, k = 0.4 and profit from evasion starting at 5.

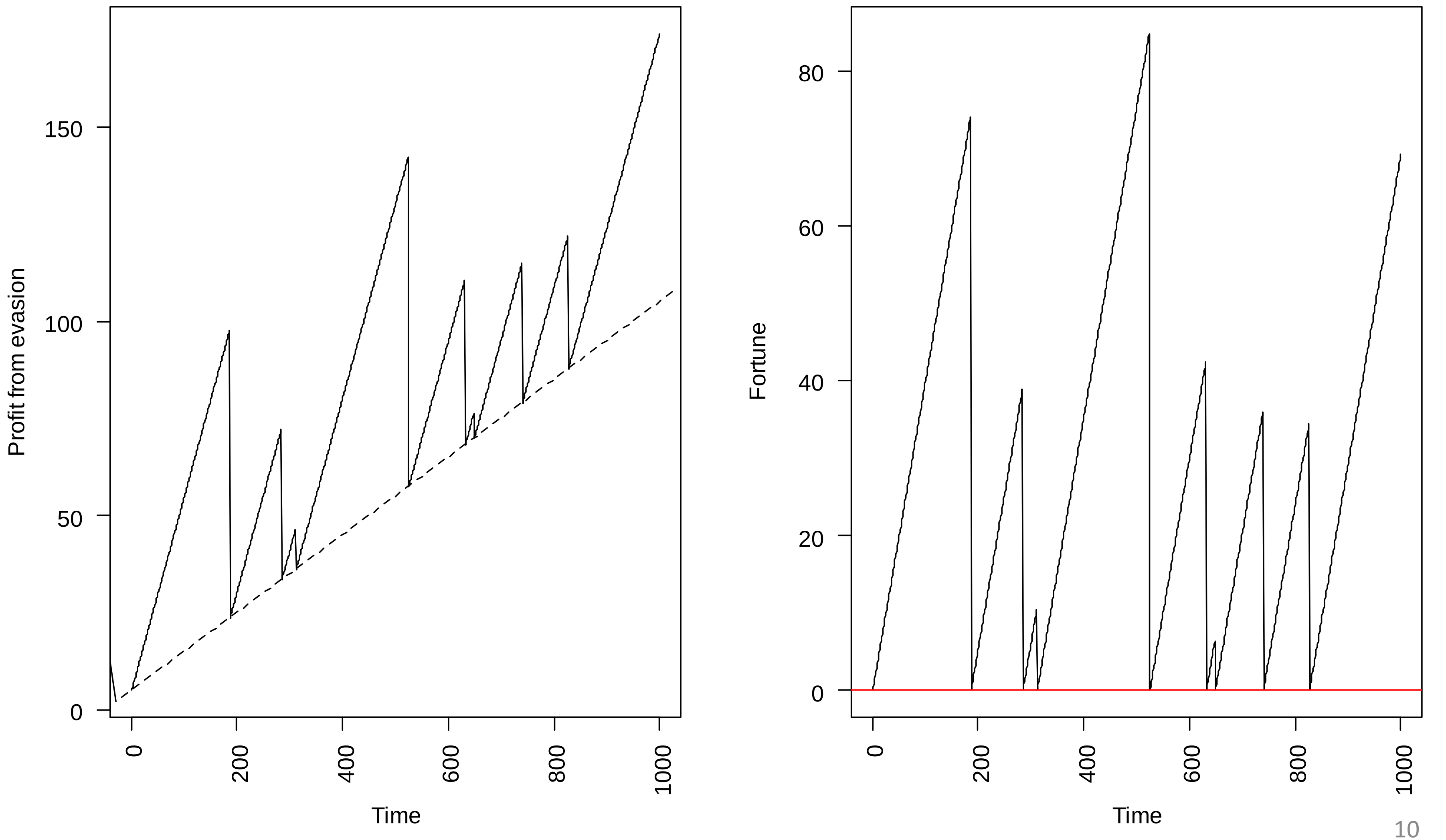
- Always evades up until some time and never evades afterwards.
- Average change in profit between evasions is constant.





An example with τ = **50%**, p = 0.01, λ = 1.5, k = 0.4 and profit from evasion starting at 5.

- Always evades.
- Average change in profit between evasions is constant.



3.2. Suppose the taxpayer is audited at time T_0 and at time $T_1 > T_0$ and there are no audits in between. We consider the total profit made from evasion

$$\mathrm{pf}(T_1) - \mathrm{pf}(T_0)$$

between T_0 and T_1.

3.5. Although T_0 and T_1 are random variables, we see that in all cases the quantity

$$\frac{\mathrm{pf}(T_1) - \mathrm{pf}(T_0)}{T_1 - T_0} = \tau - \min(k, \lambda\tau) = \begin{cases} \tau - k & \tau > k/\lambda \\ \tau(1-\lambda) & \tau \leq k/\lambda \end{cases}$$

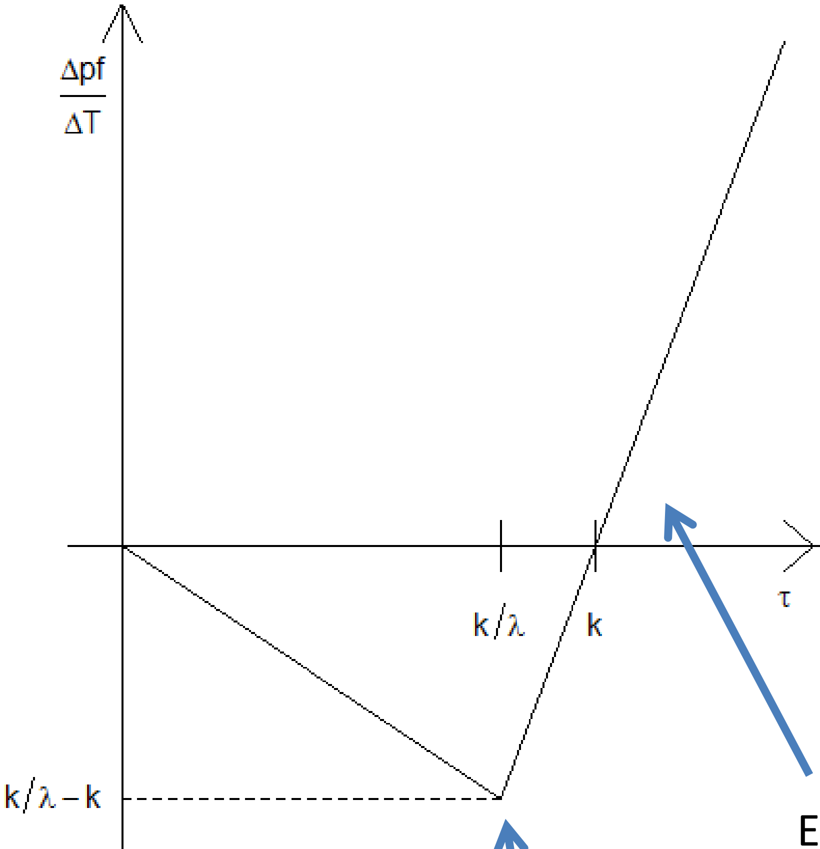
is constant. This quantity represents the average change in pf between audits. Equivalently, it represents the average profit from evasion. We will use it extensively in this paper and denote it using the derivative-like symbol $\frac{\Delta\,\mathrm{pf}}{\Delta T}$.

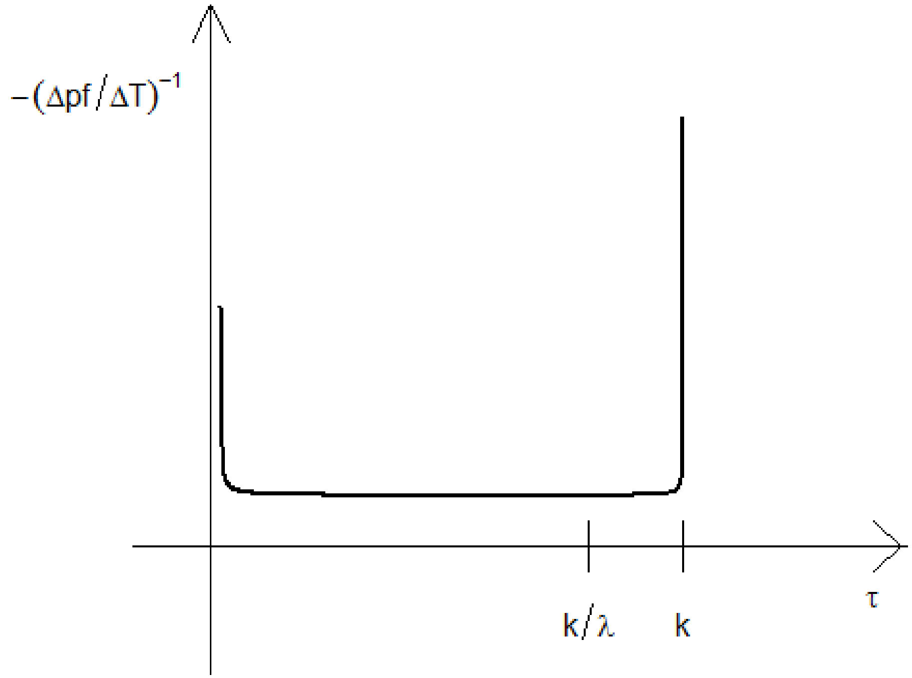
Time until compliance:

$$E[T_{\mathrm{comp}}] = -\,\mathrm{pf}(0)\left(\frac{\Delta\,\mathrm{pf}}{\Delta T}\right)^{-1} + \frac{1}{p} = \begin{cases} \frac{\mathrm{pf}(0)}{(k-\tau)} + \frac{1}{p} & \tau \geq k/\lambda \\ \frac{\mathrm{pf}(0)}{\tau(\lambda-1)} + \frac{1}{p} & \tau < k/\lambda \end{cases} \tag{3.1}$$

In particular, the expected time until the taxpayer becomes compliant is a linear function of $-\left(\frac{\Delta\,\mathrm{pf}}{\Delta T}\right)^{-1}$.

This will be the number of times the taxpayer evades until they become compliant in the case τ < k. (In the case τ > k, the taxpayer always evades anyway.)





Evasion when $\tau > k$

$\tau = k/\lambda$ is the tax rate that minimises evasion (in the sense that choosing this tax rate causes the taxpayer to stop evading in the quickest possible time, on average.)

~~**Paradox!**~~

Aside:

Link to progressive taxation (from earlier)

The optimal tax rate k/ λ is proportional to k, so the greater the proportion of income saved, the greater the tax rate should be to minimise evasion.

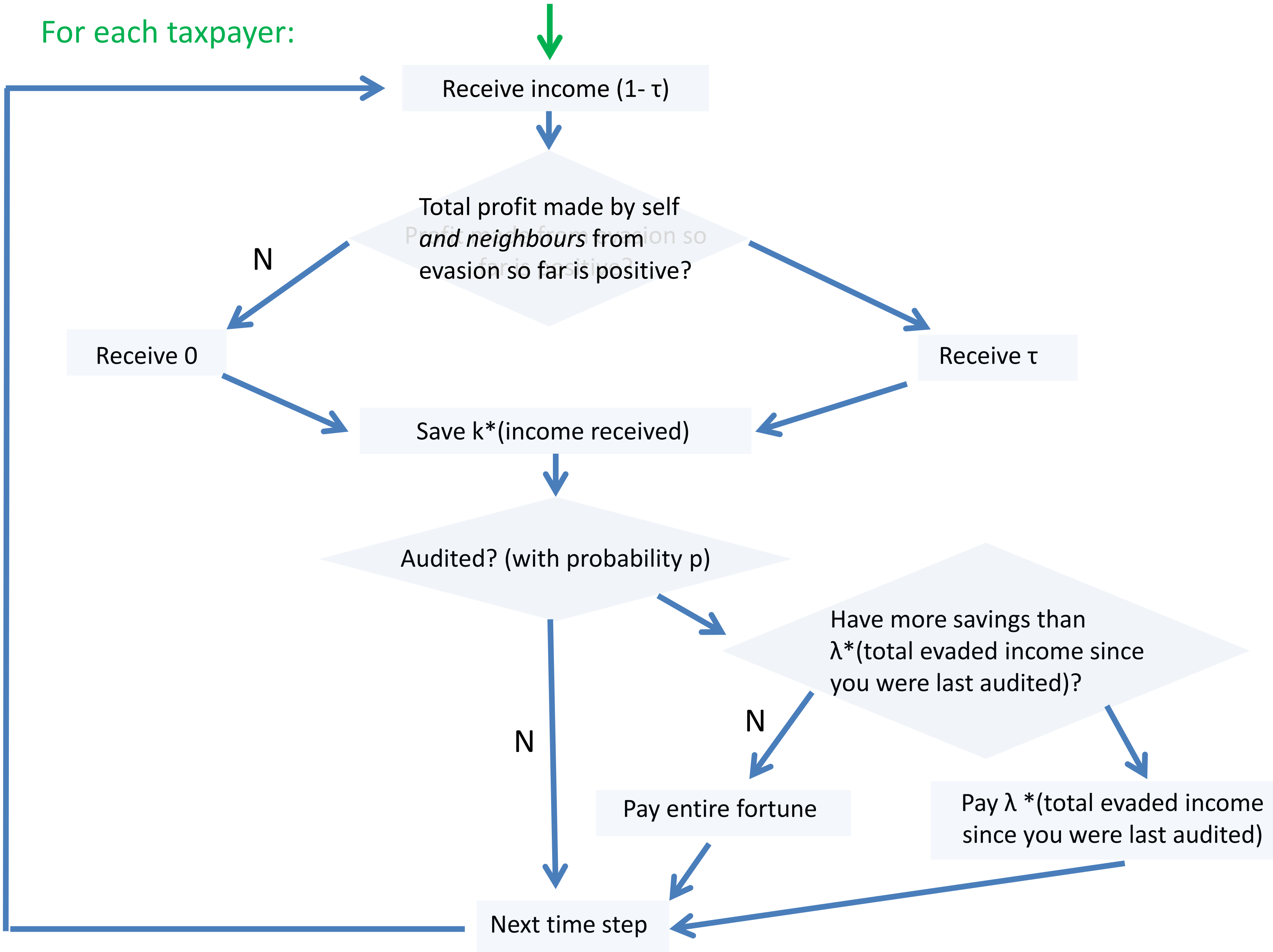
(This suggests that progressive taxation might be desirable for a different reason to the usual one.)

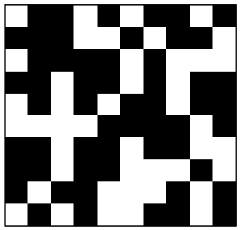
Extension: Agent-based Model

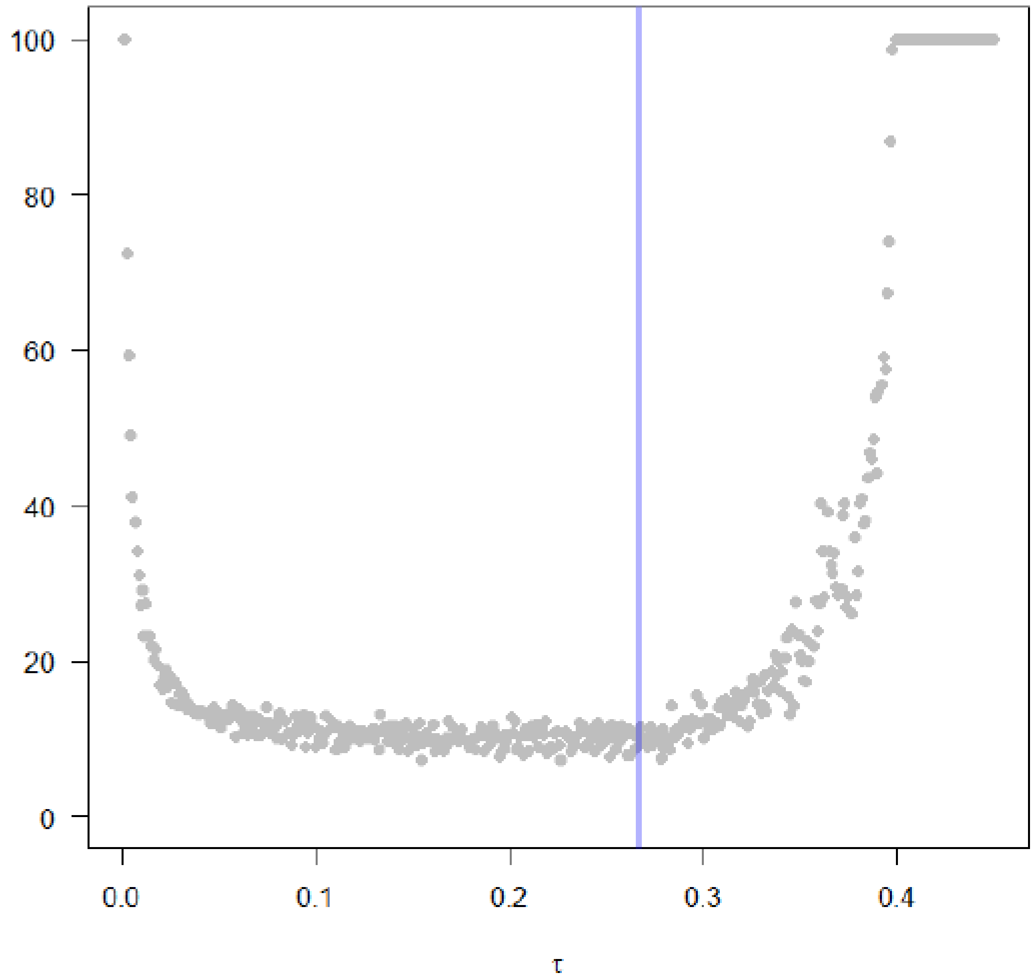
Suppose N taxpayers are connected in a network, all with the same τ, p, λ, k. We want the taxpayers to influence each other in some way and are interested in the way evasion in the network evolves over time.

(Possible application: use simulations to estimate the effect of choices of tax policy, e.g. changes in τ, p, and λ.)

For each taxpayer:







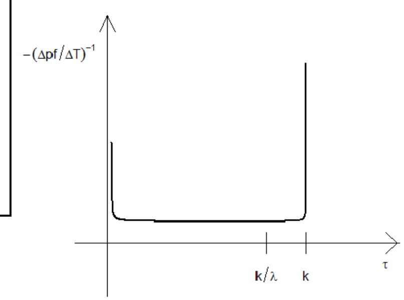
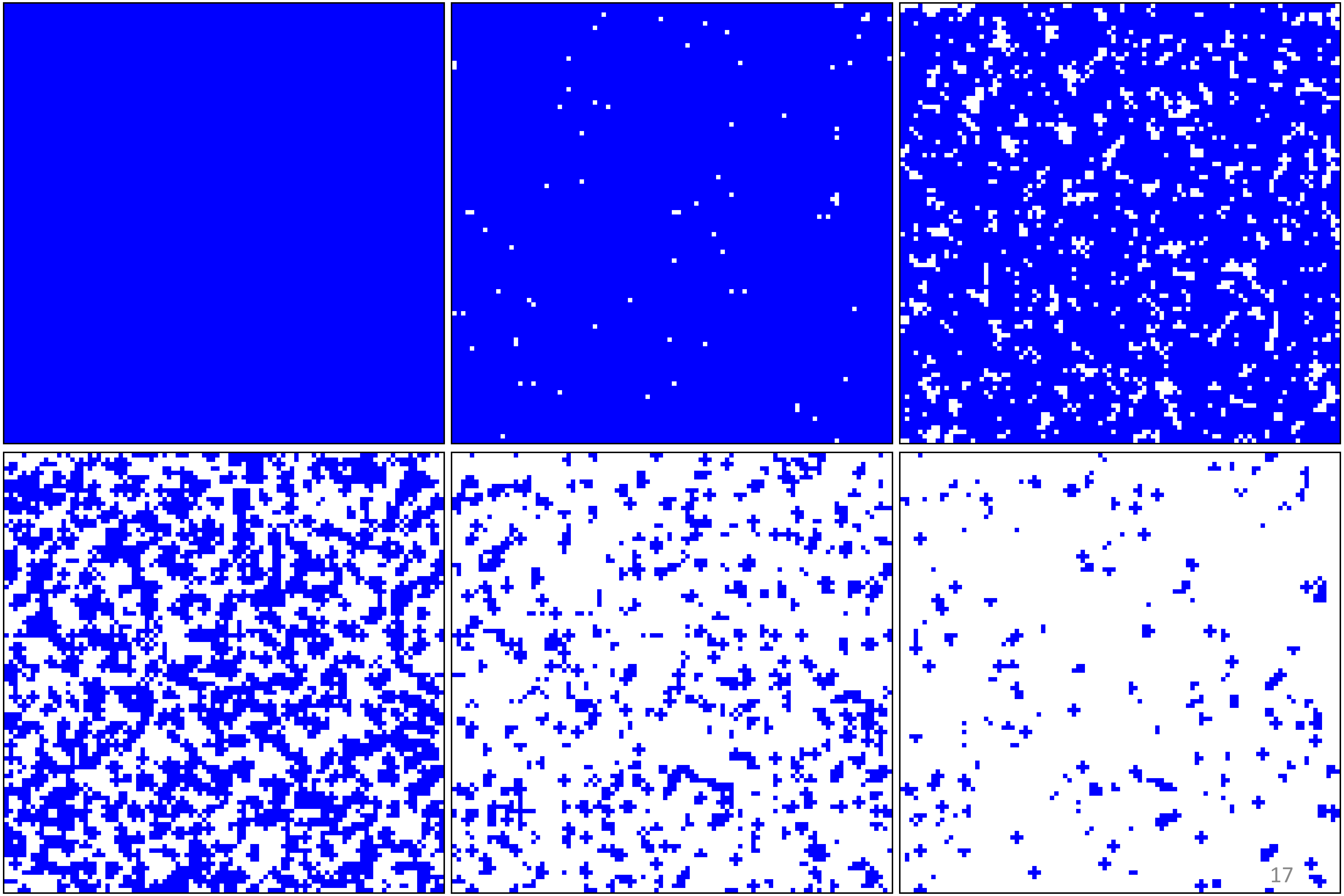
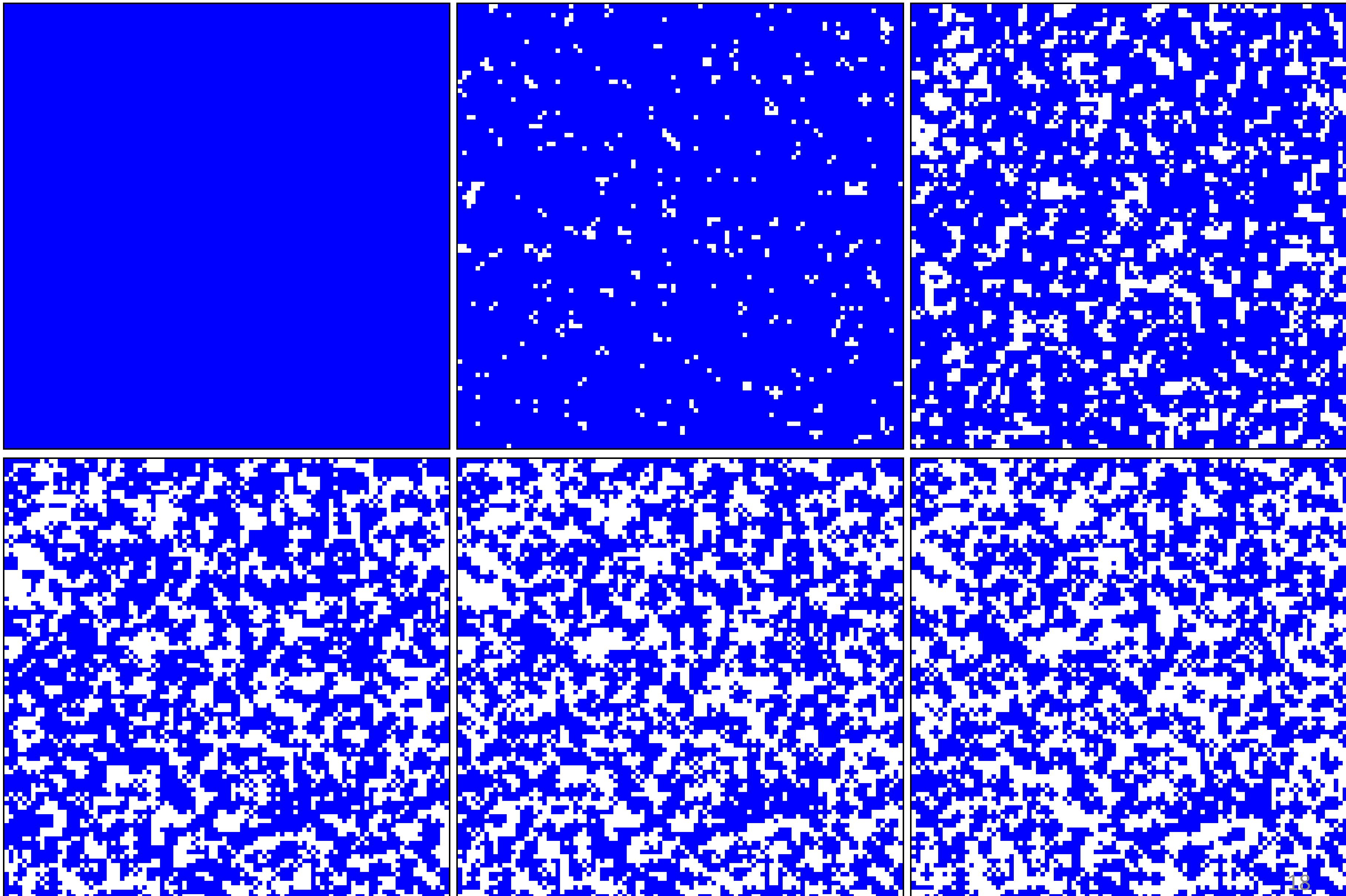


FIGURE 4. Average number of evaders versus tax rate τ for a toroidal grid with 100 taxpayers run for 1000 iterations. The line is at $\tau = k/\lambda$, the tax rate which minimised non-compliance in the one-taxpayer model.

100 x 100 toroidal grid, sampled at every 100 iterations, k = 0.4.



100 x 100 toroidal grid, sampled at every **200** iterations, **k ~ Unif(0, 0.8)**



- Network effects not that interesting.
- Consider alternatives, e.g. a different income distribution, more realistic topology…
- Although this model has some nice properties in the one-taxpayer case, it is not the *real* reason why people do/do not evade tax.