MATH 2220 HW10.

Due Wednesday 26 November (or earlier)

(1) Let a_i, b_i, c_i be real numbers. Define a vector field $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ by

$$\mathbf{F}(x, y, z) = (a_1x + a_2y + a_3z, b_1x + b_2y + b_3z, c_1x + c_2y + c_3z).$$

(a) Calculate $\mathbf{curl}(\mathbf{F})$.

(b) Calculate $\mathrm{div}(\mathbf{F})$.

(c) Find a vector field $\mathbf{F}$ such that $\mathbf{curl}(\mathbf{F}) = (3, 2, 1)$ and $\mathrm{div}(\mathbf{F}) = 7$.

(2) Let C be the boundary of the square in $\mathbb{R}^3$ with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 1)$ and $(1, 1, 1)$, oriented in the direction $(0, 0, 0) \to (1, 0, 0) \to (1, 1, 1) \to (0, 1, 1) \to (0, 0, 0)$.

Let

$$\mathbf{F}(x, y, z) = \sin(x^3)\mathbf{i} + \mathbf{j} + (e^{z^2} + 3y)\mathbf{k}.$$

(a) Calculate

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

(b) Does there exist a function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$? Explain.