MATH 2220 HW11.

Due Friday 5 December

(1) Let S be the surface obtained by rotating the graph of the function

$$y = |x| + 1$$

about the x–axis between $x = -2$ and $x = 2$.

(a) Let $\mathbf{F}(x, y, z) = (0, (x^2 - 2)z, y)$. Calculate

$$\iint_S \mathbf{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

with respect to either orientation.

(b) Let $\mathbf{F}(x, y, z) = (2x, y + z^3 e^{z^3}, x)$. Calculate the outward flux

$$\iint_{\partial B} \mathbf{F} \cdot d\mathbf{S}$$

where B is the solid region bounded by S and the planes $x = 2$ and $x = -2$.

(2) Let c be a constant and let $\mathbf{E} = \mathbf{E}_t$ and $\mathbf{B} = \mathbf{B}_t$ be time-dependent vector fields defined by

$$\mathbf{E}_t = ((t + 1)x, -ty, -z)$$

$$\mathbf{B}_t = \frac{1}{c^2}(0, 0, yx).$$

(a) Show that $\nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{B} = 0$

(b) Show that $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ and $\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$.

(c) Let S be any oriented surface. Show that

$$\iint_S \mathbf{E} \cdot d\mathbf{S} = (t + 1)c^2 \oint_{\partial S} \mathbf{B} \cdot d\mathbf{r} + \oint_{\partial S} yz\,dx.$$