MATH 2220 HW2 SOLUTIONS.

Homework 2. Due Wednesday 10 September.

(1) Section 2.1 p. 105–107

(a) # 2a.

If $c < 1$, *then the level curve is empty. If* $c = 1$ *then the level "curve" is a single point* $(0, 0)$. *If* $c > 1$ *then the level curve is a circle with midpoint* $(0, 0)$ *and radius* $\sqrt{c-1}$.

(b) # 17.

The question asks us to draw the set of points $\{(x, y, z) \in \mathbb{R}^3 : z = |y|\}$. *The sections* $x = k$ *for* $k \in \mathbb{R}$ *are the graph of* $z = |y|$ *in the* yz*–plane, for every value of* k. *So the overall shape is a cylinder with cross-section the graph of* $z = |y|$, *that is, a* $\vee$*–shape.*

(c) # 30.

This is an ellipsoid, as described in lectures. You should draw a sketch and label the points where the ellipsoid intersects the three coordinate axes. Notice that the cross-sections in the planes $y =$ *constant are circles.*

(2) Section 2.2, p. 125–127.

(a) # 8a.

If $xy \neq 0$ *then*

$$\frac{(x+y)^2 - (x-y)^2}{xy} = \frac{4xy}{xy} = 4$$

so the limit exists and equals 4.

(b) # 16b.

Oops! This is an ordinary function from $\mathbb{R}$ *to* $\mathbb{R}$. *I meant to set 17(c). Anyway,* f *is continuous because of the rules for combining continuous functions that we had in lectures. The one subtlety is that* $2 - \sin(x)$ *has to be nonzero on the whole of* $\mathbb{R}$. *This is true because* $\sin(x)$ *only takes values between* -1 *and* 1 *and so* $2 - \sin(x)$ *can never equal zero.*

(3) For each of the following sets S, state whether S is open, closed or neither. Draw a sketch of S. What is the boundary of S? (You should justify your answers, but detailed proofs are not required.)

(a) S = the set of points (x, y) in $\mathbb{R}^2$ which satisfy $x \geq 0$ and $y < 0$.

The boundary of S is $\{(x, 0) : x \geq 0\}$ together with $\{(0, y) : y \leq 0\}$. S is neither open nor closed; it contains some of its boundary points on the x–axis, but does not contain some of them on the y–axis. A rigorous proof is not required.

(b) S = the line $2x + 3y = 5$.

This was meant to be in $\mathbb{R}^2$, but the same answer works in $\mathbb{R}^3$. Here S is a closed set. This is because if P is some point not on the line then there is a point Q on the line which is closer to P than all the other points on the line. The distance $dist(P, Q)$ is positive, so if $r = dist(P, Q)/2$ then $D_r(P)$ is an open ball containing P which does not contain any point of S. Thus, the complement of S is open, so S is a closed set. The boundary of S consists of S itself (we have already shown that no point not in S can be a boundary point. Every point R of S is a boundary point because every ball centred at R must contain a point on the line S and a point not on S as well. Again, a rigorous proof is not required.)

(c) $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$. *Recall that this means: "S is the set of points (x, y) in $\mathbb{R}^2$ which satisfy $x^2 + y^2 < 1$."*

This is the open disc $D_1((0, 0))$. See the lecture notes for this example. The boundary is the circle $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$.