MATH 2220 HW9.

Due Wednesday 19 November

(1) Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the semicircle $x^2 + y^2 = 1$, $y \geq 0$, oriented anticlockwise, and $\mathbf{F}(x, y) = (-y, x)$.

(2) Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve in $\mathbb{R}^3$ given by the parametrization $c(t) = (t, t^2, t^3)$, $0 \leq t \leq 1$ and $\mathbf{F}(x, y, z) = (z, 1, x^2)$.

(3) A wire whose shape is given by the curve $(t, \log(t), t^2 - 1)$, $1 \leq t \leq 2$, is made of a material whose density at the point (x, y, z) is $f(x, y, z) = e^{2y}$. Find the mass

$$\int_C f(x, y, z) ds$$

of the wire.

(4) A triangle T with vertices $(0, 0, 0)$, $(1, 1, 1)$ and $(-1, -1, 1)$ is made of the same material as in the previous question. Find the mass

$$\iint_T f(x, y, z) dS$$

of the triangle. (Hint: an indefinite integral of ue^u is $(u - 1)e^u$.)

(5) Let B be the solid region satisfying the equations $z \geq 0$ and $1 \leq x^2 + y^2 + z^2 \leq 4$. Let S be the surface of B, oriented outward. Calculate the flux through S of the vector field $\mathbf{F}(x, y, z) = (z, 0, -1)$. (Hint: S should be split into three parts; a hemisphere of radius 1, a hemisphere of radius 2, and an annulus (ring) in the xy–plane.)