MATH 2220 PRELIM 2

You have 1 hour 30 minutes to complete this exam. The exam starts at 7:30pm. Each question is worth 20 marks. There are 5 questions in total. You are free to use results from the lectures, but you should clearly state any theorems you use. **The exam is printed on both sides of the paper.** *Good luck!*

(1) (a) Find all the critical points of

$$f(x, y) = x^2 - 6xy + 10y^2$$

and determine their nature.

(b) Find the second order Taylor polynomial of

$$g(x, y) = x^2 e^x - 6x(e^y - 1) - 20(\cos(y) - 1)$$

about the point $(0, 0)$ (you may omit the remainder term).

(2) Calculate

$$\int_0^1 \int_{\sqrt[3]{y}}^1 e^{x^4} dx dy.$$

(3) (a) The number of cookies which can be baked from x pounds of flour, y pounds of sugar and z pounds of butter is

$$C(x, y, z) = xyz + 3.$$

Assuming that we can buy a total of six pounds of ingredients, what is the maximum number of cookies which can be produced?

(b) Find the maximum and minimum values of the function $f(x, y) = x^4 + y^4$ on the disc $x^2 + y^2 \le 2$ [TURN OVER].

(4) Suppose x, y, z, w are related by the following equations.

$$xe^z + 2w + 5y = 0$$

$$w + z + yz^5 + 4w^6 = 0$$

(a) Show that these equations uniquely determine z and w as functions of x and y near the point $(0, 0, 0, 0)$.

(b) Calculate $\frac{\partial z}{\partial x}|_{(0,0)}$ and $\frac{\partial w}{\partial x}|_{(0,0)}$.

(5) A paperweight P consists of the points in $\mathbb{R}^3$ lying above the triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$ and $(0, 1, 0)$ and below the plane $z = 3x + 2y$.

(a) Calculate the volume of the paperweight.

(b) Suppose the paperweight is made of a material whose density at the point (x, y, z) is $f(x, y, z) = 6 - z$. Set up an iterated integral for the mass

$$M = \iiint_P f(x, y, z)\,dV$$

of the paperweight. Be sure to include all limits of integration, but do not attempt to evaluate the integral.

[END.]