QUIZ

1. MENTAL

Calculate the following (it is possible to do it without using pen and paper).

(1) The outward flux of the vector field $\mathbf{F}(x, y, z) = (x, 0, 0)$ through the surface of the cube $[0, 2] \times [0, 2] \times [0, 2]$ with the cube $[0, 1] \times [0, 1] \times [0, 1]$ removed.

(2) The integral

$$\oint_C (2dx + 3dy + 4dz)$$

where C is the boundary of the triangle with vertices $(0, 0, 0)$, $(5, 6, 7)$ and $(0, 0, 1)$ in $\mathbb{R}^3$.

2. SKILL

Calculate the integral

$$\iint_D (x^2 + y^3 + y^4)dA$$

where D is the semicircle $x^2 + y^2 \leq 1$, $y \geq 0$ in the xy–plane.

3. PHYSICAL

The gravitational field of a planet with mass M centered at $(0, 0, 0)$ is given by

$$\mathbf{F} = -\frac{GM\mathbf{r}}{\|\mathbf{r}\|^3},$$

where $\mathbf{r} = (x, y, z)$.

(1) Calculate the work done by $\mathbf{F}$ on a spacecraft which makes one complete orbit of the planet, following the circle $x^2 + y^2 = 100$, $z = 0$ from $(100, 0, 0)$ to $(100, 0, 0)$ in the anticlockwise direction.

4. MYSTERY

Consider the two circles C_1 and C_2 in $\mathbb{R}^3$, where C_1 is given by the equations $x^2+y^2=1$ and $z=0$, and C_2 is given by the equations $(x-5)^2+z^2=1$ and $y=0$. Let $\mathbf{F}=(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0)$.

It is a fact that $\mathbf{curl}(\mathbf{F})=\mathbf{0}$. (You can check.)

It is also true that $\int_{C_1} \mathbf{F}\cdot d\mathbf{r}=2\pi$ and $\int_{C_2} \mathbf{F}\cdot d\mathbf{r}=0$. (You can check.)

Bob claims that because $\mathbf{curl}(\mathbf{F})=\mathbf{0}$, if there is a surface S with boundary consisting of the circles C_1 and C_2 then we must have

$$\iint_S \mathbf{curl}(\mathbf{F}) = 0 = \pm\int_{C_1} \mathbf{F}\cdot d\mathbf{r} \pm \int_{C_2} \mathbf{F}\cdot d\mathbf{r} = \pm 2\pi.$$

because of Stokes' Theorem. Therefore, there cannot be any surface whose boundary consists precisely of the circles C_1 and C_2. Sally, on the other hand, argues that you could obviously have a pipe with its ends being the circles C_1 and C_2, and therefore such an S *does* exist. Who is right?