**MATH 4130 HONORS INTRODUCTION TO ANALYSIS I. PRELIM 1. THURSDAY MARCH 11 2010**

**Please attempt all questions. You have 70 minutes. You may use any theorems from the lecture notes, but please clearly state any theorems which you use.**

(1) **(9 marks)** Let $X = (0,1) \cup [3,4] \subset \mathbb{R}$. State whether the following statements about $X$ are true or false and give a brief reason in each case.
   (a) $\sup(X) = 4$.
   (b) $X$ can be written as a union of open sets.
   (c) $|X| = |\mathbb{R}|$.

(2) **(19 marks)** Let $\{x_n\}$ be a sequence of real numbers.
   (a) **(3 marks)** State what it means for $\{x_n\}$ to *converge* to the limit $L \in \mathbb{R}$.
   (b) **(8 marks)** Let $k \in \mathbb{N}$ and define a sequence $\{y_n\}$ by $y_n = x_{n+k}$, $n \geq 1$. Suppose $\{x_n\}$ converges to $L$. Show that $\{y_n\}$ also converges to $L$.
   (c) **(8 marks)** Let $x_1 \in \mathbb{R}$ and define a sequence of real numbers $\{x_n\}$ by

$$x_{n+1} = x_n^2 + x_n + 1, \qquad n \geq 1.$$

   Show that the sequence $\{x_n\}$ does not converge.

(3) **(22 marks)** Let $A \subset \mathbb{R}$.
   (a) **(3 marks)** Explain what it means to say that $x \in \mathbb{R}$ is a *cluster point* (a.k.a. limit-point; accumulation point) of $A$.
   (b) **(3 marks)** Explain what it means to say that the set $A$ is *bounded*.
   (c) Now let

$$S = \{x \in \mathbb{R} : x \text{ is a cluster point of } A\}.$$

      (i) **(8 marks)** Show that $S$ is a closed set.
      (ii) **(8 marks)** Suppose $A$ is bounded. Show that $S$ is a compact set.

**[END OF PAPER]**